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LETTER TO THE EDITOR

Evidence of self-organized criticality in dry sliding friction

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Abstract

This letter presents experimental results on unlubricated friction, which suggests that stick–slip is described by self-organized criticality (SOC). The data, obtained with a pin-on-disc tribometer examines the variation of the friction force as a function of time—or sliding distance. This is the first time that standard tribological equipment has been used to examine the possibility of SOC. The materials were matching pins and discs of aluminium loaded with 250, 500 and 1000 g masses, and matching M50 steel couples loaded with a 1000 g mass. An analysis of the data shows that the probability distribution of slip sizes follows a power law. In addition, the frequency power spectrum follows a $1/f^{\alpha}$ pattern with α in the range 1.1–1.3. We perform a careful analysis of all the properties, beyond the two just mentioned, which are required to imply the presence of SOC. Our data strongly support the existence of SOC for stick–slip in dry sliding friction.

1. Introduction

The subject of the relationship between stick–slip in dry sliding friction and self-organized criticality (SOC) has raised controversy in the last few years [1]. On the one hand there are experimental and theoretical studies, other than friction, that suggest that a many-degree of freedom system far from equilibrium organizes naturally in a critical state, releasing energy through fast relaxation events (avalanches) of different sizes, these sizes being distributed according to a power-law probability density. An example is a pile of sand [2] in which grains of sand are continuously introduced at its apex. When the slope of the side of the pile reaches a threshold maximum angle, a critical state, energy is released in the pile through avalanches that keep the angle constant while moving additional material from the apex to the bottom of the pile. Similar studies have been performed on earthquakes [3, 4], biological systems [5], the stock market [6], rainfall [7] and friction [8–10]. On the other hand, there have been

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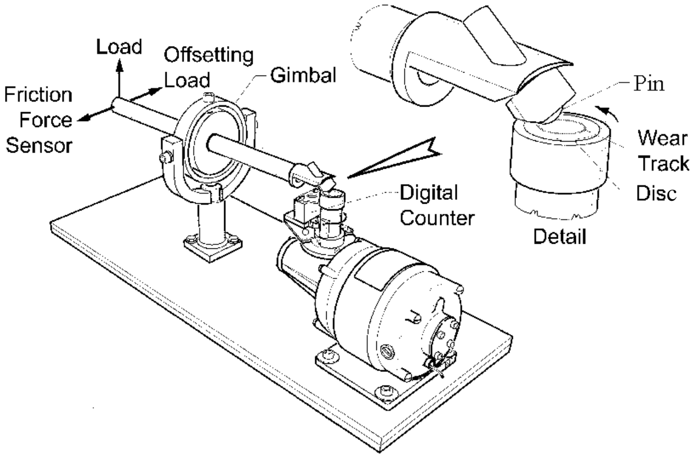

Figure 1. Pin-on-disc tribometer. The arrow points at the location where the pin and the disc touch. The disc lies horizontally while the ball, attached to the arm, rests above it.

arguments proposing that power-law behaviour may not necessarily be a sufficient condition for SOC [11, 12]. Various, more stringent, requirements have been devised to narrow the possible existence of SOC. Among them, the most relevant is the presence of a $1/f^{\alpha}$ frequency power spectrum, and the presence of a stationary state at large times, signalling the presence of an attractor which defines the critical state. In the present study, we examine stick–slip in dry friction using a pin-on-disc tribometer ordinarily used in friction and wear studies. The probability distributions of slip sizes for matching aluminium and M50 steel pins and discs are examined for evidence of SOC.

2. Experimental details

The pin-on-disc tribometer used for these experiments is shown in figure 1. This configuration was chosen because it allows for easy replacement of the contacting surfaces, and it is a standard method for measuring friction and wear in unlubricated and lubricated contacts. The apparatus uses a 2.54 cm diameter disc and a pin with a 0.95 cm radius machined on its end.

The pin is attached to a load arm that is mounted on a gimbal supported at the centre through which a load applied at the end of the arm is transferred to the contact zone. A strain gauge is mounted at the end of the arm to monitor the tangential (friction) force.

The tangential force is monitored at a sample rate of 1000 data points s^{-1} and conversion is done using a 16-bit DAQ card controlled by LabVIEW. Data are recorded to a text file for later analysis.

In order to drive the system very slowly away from static equilibrium, the rotational speed used was the slowest that the apparatus allowed (10–20 rpm). Each disc was used for up to four tests by changing the radial position of the pin on the disc and rotating the pin to give a new surface. Load masses ranged from 250 to 1000 g. Ball materials were M50 steel and aluminium. Disc materials tested were M50 steel and aluminium, and matched the ball material for a given test. The contact between ball and disc was unlubricated.

Steel was studied with a 1000 g mass applying the normal load between pin and disc. For aluminium, which is softer than steel, the load masses were 250, 500 and 1000 g to examine whether load-induced effects were present. In all cases, data were collected in sets of 10^6 points. The first 4 min of each data set was discarded to ensure that a steady state was reached.

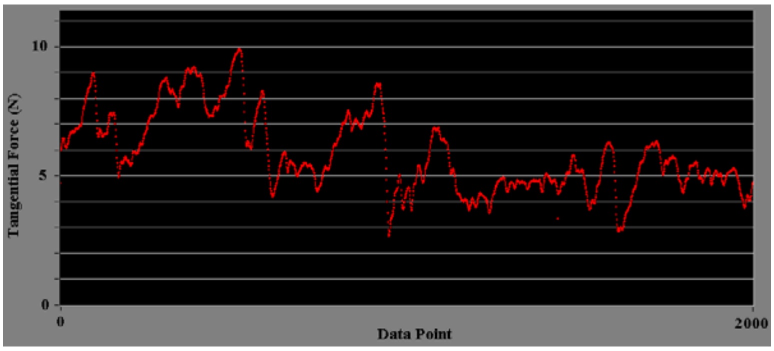

Figure 2. Typical signal from the tribometer. The effective spring constant of the apparatus was $\sim 1\ \mu\mathrm{m\ g}^{-1}$, giving the largest slips as a few hundred microns.

(This figure is in colour only in the electronic version)

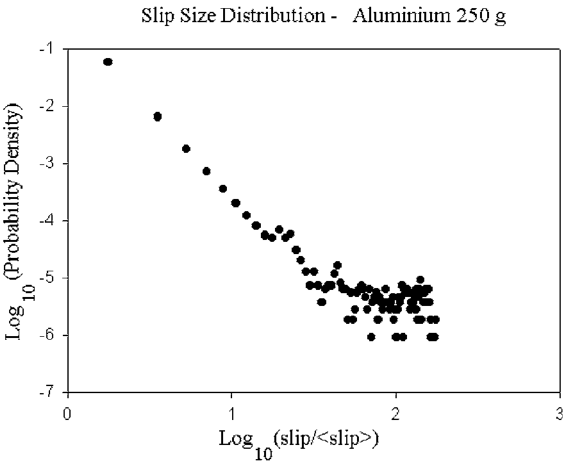

Figure 3. Probability density of jump sizes for aluminium with a 250 g load.

3. Results

Figure 2 shows typical results for tangential (friction) force versus time. Slips of various sizes are observed. The first, elemental test of SOC regards the probability distribution of slip sizes, which must follow a power law in order to have an SOC state. We present a typical result of such an analysis for aluminium with a 250 g load in figure 3. The linear behaviour with negative slope is apparent for four to five decades, giving a power law with exponent 3.17. The data scatter at high slip sizes is due to poor statistics. There is a small bump in the curve a little above $\frac{\mathrm{slip}}{\langle\mathrm{slip}\rangle} = 1$. It corresponds to the systematic once-per-revolution wobble of the disc surface, due to residual misalignment of the surface perpendicular to the axis of rotation. That wobbling manifests as artificial slips of about 100 g. For aluminium with a load of 250 g, the average slip size, was measured to be $\langle\mathrm{slip}\rangle \approx 4$ g, therefore the effect should be noticeable at an abscissa value of ~ 1.4 ($\approx \log_{10}(100/4)$) as seen in figure 3.

Similar results were obtained for higher loads and for steel, and are shown in table 1. We were interested in examining whether the exponent of the power law might depend on load. Figure 4 shows the exponents (for example the slope of the curve in figure 3) versus load for

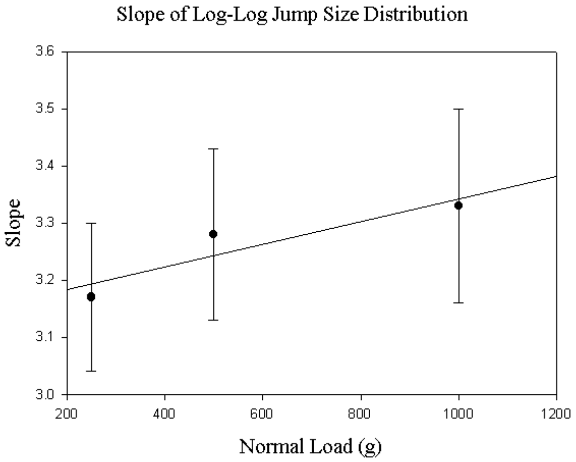

Figure 4. Slope of the log–log curve of the probability density of jump sizes, as a function of normal load in aluminium samples.

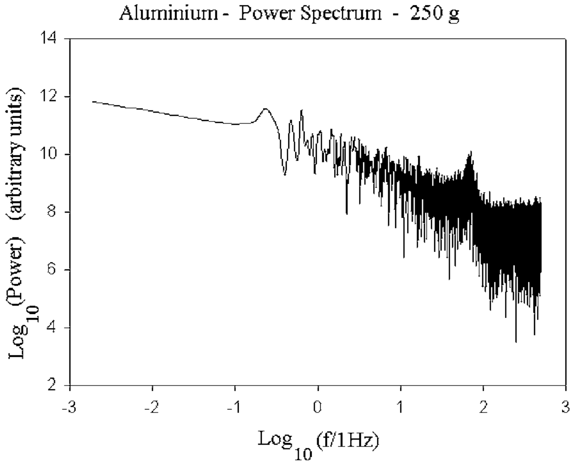

Figure 5. Frequency power spectrum of the original data for aluminium at 250 g load.

aluminium. The figure suggests that there could be an upward trend between exponent and load. However, within the uncertainty of the measurement, no hard conclusion can be drawn. In fact, a constant, horizontal line could be drawn through the experimental results in figure 4.

Thus, our data support a power-law dependence with an exponent in the range 3.0–3.5 for aluminium. For steel with a 1000 g load the exponent is 2.83. This suggests that steel at high loads might behave similarly to aluminium at smaller loads, which could be expected from a naïve analysis of the number of asperities in contact for a given load for each material.

As mentioned, a power-law distribution of slip sizes is not sufficient to claim SOC. Therefore, we also analysed the frequency content of the data. Figure 5 shows a typical frequency power spectrum of the original data for aluminium at 250 g load, by taking the Fourier transform of $2^{19} = 524\,288$ data points in time and squaring the result. The curve is linear with a negative slope at low and medium frequencies. At high frequencies, however,

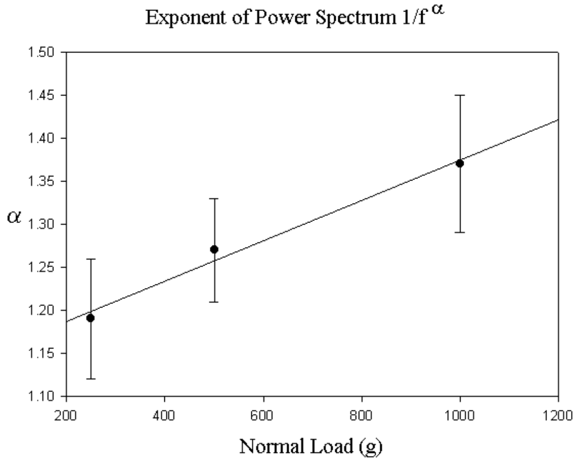

Figure 6. Exponent of the power spectrum versus load.

Table 1. Exponent of relevant curves for aluminium at various loads.

Critical exponents	Frequency power spectrum	Slip size distribution
250 g, aluminium	1.19 ± 0.07	3.17 ± 0.13
500 g, aluminium	1.27 ± 0.06	3.28 ± 0.15
1000 g, aluminium	1.37 ± 0.08	3.33 ± 0.17
1000 g, steel	1.16 ± 0.06	2.83 ± 0.12

the spectrum looks like white noise. This can be explained as follows. Before performing the measurements, we checked the system to quantify the noise level of unknown origin, most likely caused by mechanical vibrations of heavy machinery in the building. We found that the maximum amplitude of that random noise corresponded to about 1 g.

On the other hand, when the pin and disc were in contact, typical large slips were of the order of a few hundred grams. Thus, the low amplitude, high frequency part of the spectrum in figure 5 corresponds to noise of undetermined origin. Also, notice the peak at about 60 Hz. All these features were present in all the data sets analysed. A linear regression fit to the data in figure 5 up to $f \approx 20$ Hz behaves as $1/f^{1.19}$. Similar analysis was performed for aluminium at other loads and for steel (see table 1). We plot the exponent of the power spectrum versus load in figure 6. The upward trend is more noticeable than in figure 4, suggesting a possible load dependence.

Again, in table 1 it can be seen that steel at high loads behaves similarly to aluminium at small loads.

4. Discussion

Pin-on-disc, dry sliding stick–slip thus satisfies the two most important characteristics for SOC. Is stick–slip in dry friction therefore an example of SOC? One concern is that the number of states in the system may not be large enough for self-organization, since the typical number of asperities making a contact is classically assumed to be in the few hundreds [13]. However, recent theoretical [14, 15, 20] and experimental [16, 17] microscopic studies show stick–slip behaviour even in the presence of a single asperity, which suggests that microasperities may be the determining mechanism, and thus a smaller scale may be dominant. Nevertheless, whatever the mechanism, friction has its origin at a microscopic level, where a large number of atoms or clusters of atoms play a role [20]. Another issue of concern is whether a critical state exists, i.e. whether the system in steady state has reached an attractor. To answer this question we analysed the data for all cases in different segments, after the 4 min transient, and consistently obtained the same results for a given material and load, suggesting that the results are independent of initial conditions. Finally, we note that, unlike other experimental studies, we have not found any finite-size effects in the power law. We believe this is because the friction force in our equipment is localized to the contact area of the pin after run-in. Thus, the results do not vary with small variations in the dimensions of the pin. As a final check, we studied the distribution of waiting-times between slips again obtaining a power law, since it has been suggested this was also a necessary condition for SOC [18]. However, it has also been argued that this distribution will always be a power law for events larger that a certain size [19]. Therefore, this test cannot be conclusive due to experimental resolution limits.

At this point we cannot offer a specific theoretical model based on solid state considerations, since there are no such accepted models for either friction [1] or SOC [20]. In addition, as indicated both theoretically [21] and experimentally [17] we are dealing in complex effects at the atomic scale even in a single-asperity contact. Finally, we believe that within the limits of the experimental uncertainties, and to the extent that SOC and dry friction are understood today, stick–slip in dry friction behaves as an SOC system. This study implies the clear need for further research into SOC in tribology.

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